Critical Peaks Redefined

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Okui's confluence criterion

Theorem (Okui 1998)

a left-linear first-order term rewrite system is confluent if multi–one critical peaks $s \mathrel{\leftarrow\!\!\circ\!\!-} t \rightarrow u$ *are many–multi joinable* $s \twoheadrightarrow w \mathrel{\leftarrow\!\!\circ\!\!-} u$

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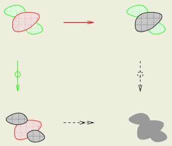
Proof outline.

1. $\mathrel{\leftarrow\!\!\!\circ\!\!\!-} \cdot \to \subseteq \twoheadrightarrow \cdot \mathrel{\leftarrow\!\!\!\circ\!\!\!-}$ by de/recomposing (needs term structure)
2. $\mathrel{\leftarrow\!\!\!\circ\!\!\!-} \cdot \twoheadrightarrow \subseteq \twoheadrightarrow \cdot \mathrel{\leftarrow\!\!\!\circ\!\!\!-}$, by 1 (trivial induction, abstract)
3. $\twoheadleftarrow \cdot \twoheadrightarrow \subseteq \twoheadrightarrow \cdot \twoheadleftarrow$, by 2 (abstract, using $\to \subseteq \mathrel{-\!\!\!\circ\!\!\!\to} \subseteq \twoheadrightarrow$) $\square$

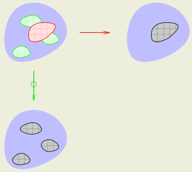
Okui's confluence criterion, pictorially

Theorem



then confluent

Proof.



multi–one peak

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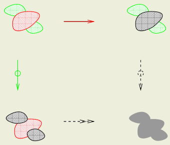

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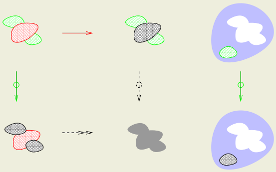

decompose into critical and empty peak

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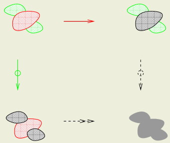

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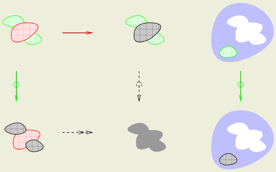

many–multi joinability by assumption

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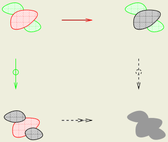
Theorem



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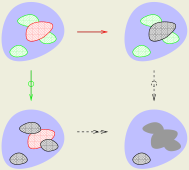

many–multi joinability by recomposition

□

Okui's confluence criterion, higher-order?

- extension to Nipkow's higher-order pattern rewrite systems?

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- renewed interest because of co-authors (formalisation, tools)

Integrating confluence-by-critical-pair criteria

Theorem (Huet)

term rewrite system is locally confluent if all critical pairs joinable

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Abstract rewrite systems integration

Newman's Lemma and diamond property: decreasing diagrams

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Abstract rewrite systems integration

Newman's Lemma and diamond property: decreasing diagrams

Term rewrite systems integration

driven by re/decomposition with critical peaks as base case
Birkhoff to bridge geometric and inductive (patterns)

Critical peak lemma

Lemma (critical peak)

a multi–multi peak either

- *is empty or critical; or*
- *can be decomposed into smaller such peaks*

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Assumption

- P set of multi–multi peaks closed under decomposition
- V set of valleys closed under (re)composition

Critical peak lemma

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Assumption

- P set of multi–multi peaks closed under decomposition
- V set of valleys closed under (re)composition

Theorem

if empty and critical peaks in P are in V, then all peaks in P are.

Proof.

by induction on size, using the assumption in the base case, and closure under decomposition and composition in the step case. □

De/recomposition in action

TRS

$$
\begin{array}{rclrclrcl}
a & \to & b & g(a) & \to & c & b & \to & d \\
f(g(x),y) & \to & h(x,y,y) & f(c,y) & \to & h(b,y,y) & & &
\end{array}
$$

Example (types of rewriting)

rewriting from term $t = g(f(g(a),a))$

- empty: $t = t$;
- one$^{=}$: $t \to g(f(g(b),a))$, $t \to g(f(c,a))$, $t \to g(h(a,a,a))$
- parallel: $t \mathrel{-\!\!\Vert\!\!\to} g(f(g(b),b))$, $t \mathrel{-\!\!\Vert\!\!\to} g(f(c,b))$
- multi: $t \mathrel{-\!\!\circ\!\!\to} g(h(b,a,a))$, $t \mathrel{-\!\!\circ\!\!\to} g(h(a,b,b))$
- many: $t \twoheadrightarrow g(f(g(d),a))$

De/recomposition in action

TRS

$$\begin{array}{rclrclrcl} a & \to & b & g(a) & \to & c & b & \to & d \\ f(g(x),y) & \to & h(x,y,y) & f(c,y) & \to & h(b,y,y) & & & \end{array}$$

Example (de/recomposing peaks)

multi–parallel peak $g(h(b,a,a)) \mathrel{\leftarrow\!\circ} g(f(g(a),a)) \mathrel{+\!\!\!\!\!\!\rightarrow} g(f(c,b))$

- empty peak $g(z) = g(z) = g(z)$; empty joinable
- multi–parallel peak $h(b,a,a) \mathrel{\leftarrow\!\circ} f(g(a),a) \mathrel{+\!\!\!\!\!\!\rightarrow} f(c,b)$
  - empty–one peak $a = a \to b$; one–empty joinable
  - critical multi–one peak $h(b,u,u) \mathrel{\leftarrow\!\circ} f(g(a),u) \to f(c,u)$; empty–one joinable (by rule $f(c,y) \to h(b,y,y)$)

  parallel–one joinable $h(b,a,a) \mathrel{+\!\!\!\!\!\!\rightarrow} h(b,b,b) \leftarrow f(c,b)$

parallel–one joinable $g(h(b,a,a) \mathrel{+\!\!\!\!\!\!\rightarrow} g(h(b,b,b)) \leftarrow g(f(c,b))$

Corollaries to critical peak lemma

Corollary (Huet)

term rewrite system is locally confluent if all critical pairs joinable

Proof.

- P = set of all one$^=$–one$^=$ peaks
- V = set of all valleys

base case empty or ordinary (one–one) critical peak □

Corollaries to critical peak lemma

Corollary (Rosen)

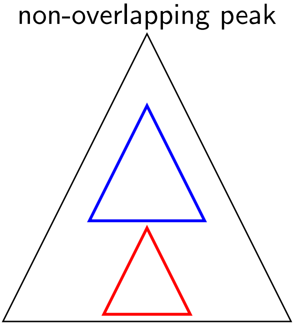
left-linear term rewrite system is confluent if it has no critical pairs

Proof.

- $P =$ set of all multi–multi peaks
- $V =$ set of all multi–multi valleys

only empty base case by assumption □

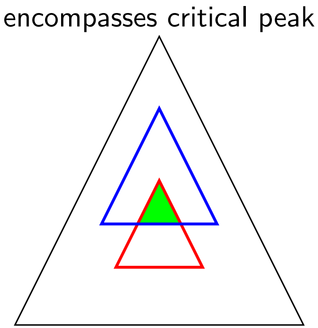
Pattern overlap intuition



Example

$a \leftarrow f(g(g(b))) \rightarrow f(g(c))$ for $f(g(x)) \rightarrow a$ and $g(b) \rightarrow c$

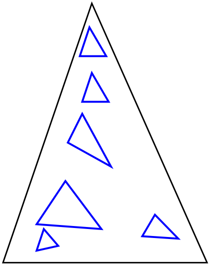
Pattern overlap intuition



Example

$h(a) \leftarrow h(f(g(b))) \rightarrow h(f(c))$ for $f(g(x)) \rightarrow a$ and $g(b) \rightarrow c$

Multiple patterns



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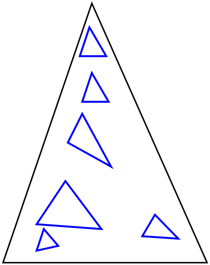
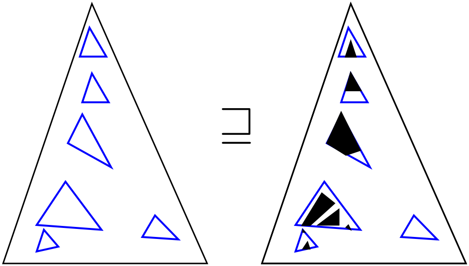


Definition (cluster)

term with multiple occurrences of patterns $t = M^{[\![\vec{X} := \vec{\ell}]\!]}$

- M is the skeleton; term linear in $\vec{X}$
- $\vec{X}$ is list of second-order variables; gaps
- $\vec{\ell}$ is list of patterns; non-var, linear first-order terms

Coarsening/refining clusters



coarser than order ⊒ (finer than ⊑) intuition: split and forget

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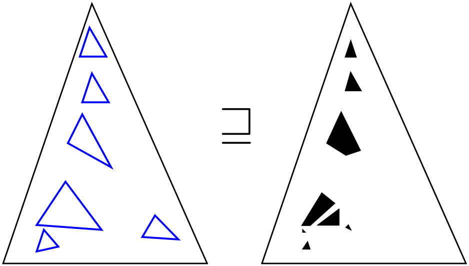
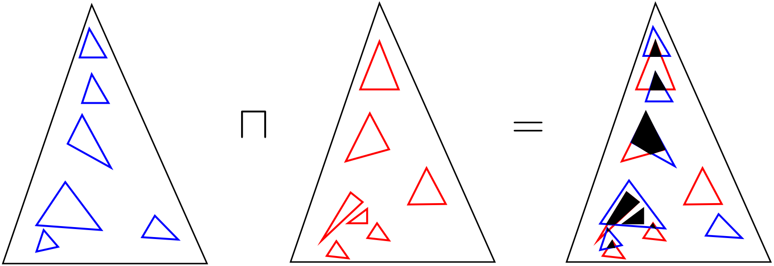


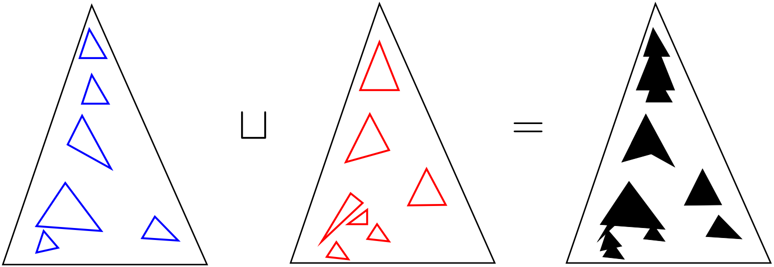
coarser than order $\sqsupseteq$ (finer than $\sqsubseteq$) intuition: split and forget

Meet of clusters



refinement order: $\varsigma \sqsubseteq \zeta$ iff $\varsigma = \varsigma \sqcap \zeta$

Join of clusters



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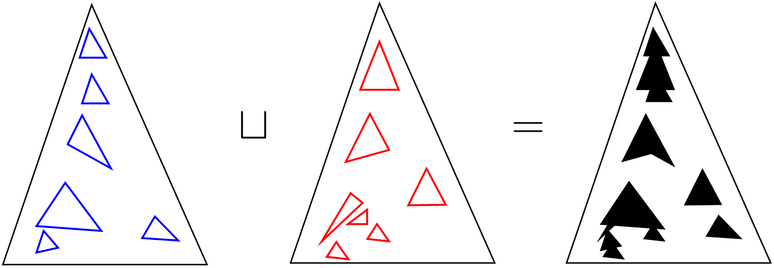

refinement order: $\varsigma \sqsubseteq \zeta$ iff $\varsigma \sqcup \zeta = \zeta$

$\bot$: term without patterns

$\top$: term one big pattern (except for root-edge, vars)

Definition

$(N,\beta) \sqsupseteq (M,\alpha)$ if $N^\gamma = M$ and $\beta = \alpha \circ \gamma$ for meta-substitution γ

Coarsening finite distributive lattice

Birkhoff's Fundamental Theorem for Distributive Lattices

a finite distributive lattice $\sqsubseteq$ is isomorphic to the $\subseteq$-lattice of downward closed sets of its join-irreducible elements

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node and edge positions are join-irreducible w.r.t. $\sqsubseteq$

Theorem

clusters are sets of positions that are downward-closed
(edge is larger than its endpoints/nodes)
$\sqsubseteq$ is finite distributive lattice isomorphic to $\subseteq$ (on sets of positions)

Redefining critical peaks via refinement

Lemma (Multisteps as clusters)

$t \multimap s$ iff $t = M^{[\![\vec{X}:=\vec{\ell}]\!]}$ and $M^{[\![\vec{X}:=\vec{r}]\!]} = s$, for rules $\overrightarrow{\ell \to r}$

Redefining critical peaks via refinement

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refinement extended to multisteps via left-hand side (t)

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Critical peak lemma

if $s\ {}_{\Phi}\!\leftarrow\!\!\multimap\ t \multimap\!\!\rightarrow_{\Psi} u$ then

- $\Phi \sqcup \Psi = \top$: empty or variable-instance of critical peak; or
- $\Phi \sqcup \Psi \neq \top$: $\Phi = \Phi_0^{[x:=\Phi_1]}$ and $\Psi = \Psi_0^{[x:=\Psi_1]}$, both smaller

Redefine?

Quote: G.-C. Rota (click)

Anyone who comes up with a new definition is likely to make enemies. No one wants to be told to drop what he or she is doing and start paying attention to the intrusion of foreign ideas.

But were critical pairs uniquely defined?

Given rules $\ell_0 \to r_0$ and $\ell_1 \to r_1$

$r_0^\sigma \leftarrow \ell_0^\sigma = C^\sigma[\ell_1^\tau] \to C^\sigma[r_1^\tau]$

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- Terese (2003): chiasmus, inner–outer, mgci

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Lemma

Critical peak equivalent to definition from literature up to chiasmus, inner,outer-order, renaming of variables, trivial peaks.

Okui revisited

Corollary (Okui)

if multi–one critical peaks are many–multi joinable then confluent

Proof.

- P = set of all multi–one$^=$ peaks
- V = set of all many–multi valleys □

Okui revisited, higher-order?

Claim

clusters of (still linear) higher-order linear patterns [Miller] are finite distributive lattice isomorphic to sets of positions with binding-info

Corollary

if multi–one critical peaks are many–multi joinable then confluent

Example

- $\beta\eta$ (with Ω)
- Carraro and Guerrieri's call-by-value λ-calculus (`759.trs`)

```
app (emb (abs (\x. M x))) (emb V) -> M V,
app (app (emb (abs \x. M x)) N) L ->
  app (emb (abs \x.app (M x) L)) N,
app (emb V) (app (emb (abs \x. M x)) N) ->
  app (emb (abs \x. app (emb V) (M x))) N
```

More consequences of critical peak lemma

Corollary (Gramlich,Toyama,Felgenhauer)

confluent if parallel–one critical peaks are many–parallel joinable

Proof.

- P = set of all parallel–one$^=$ peaks
- V = set of all many–parallel valleys

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More consequences of critical peak lemma

Corollary (Huet, Toyama, vO)

confluent if every inner–outer critical peak multi–empty joinable

Proof.

- P = set of all multi–multi peaks
- V = set of all multi–multi valleys □

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- critical peak redefinition as $\Phi \sqcup \Psi = \top$

Conclusion

- integrated critical peak criteria
- based on de/recomposition with critical peaks as base case
- refinement is finite distributive lattice on clusters
- positions synthesised (via Birkhoff) as join-irreducible clusters
- critical peak redefinition as $\Phi \sqcup \Psi = \top$
- critical peak definitions in literature all covered by same def
  - one–one: Knuth–Bendix, Huet
  - parallel–one: Toyama, Gramlich
  - multi–one: Okui
  - multi–multi: Felgenhauer

RFC

- what is a/the good definition of critical peak (and why)?
  (definitions of critical pair in literature all distinct;
  even: $\{f(x) \to x, f(y) \to y\} = \{f(z) \to z\}$?)
- integration of Huet (critical pair lemma) and Rosen (ortho)?
- why first-order rewriting defined via contexts/substitutions?
- 2nd-order definition via encompassment of 1st-order rewriting?
- node/edge positions? to be avoided?
- refinement lattice of clusters in first-/higher-order?
- why higher-order theory/tools seldomly used in λ-calculi?
  (often presented using undefined notion of critical peak)
- formalisation?

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  (same authors; work done at moment of FSCD deadline . . .)

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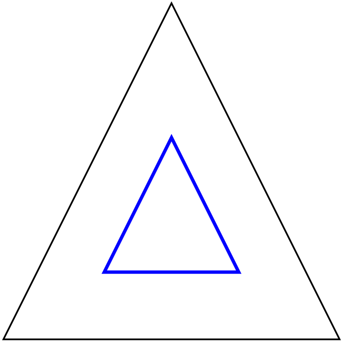
- extend to graph rewriting

Graph



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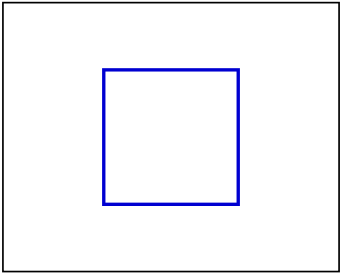

e.g. port-graph rewriting

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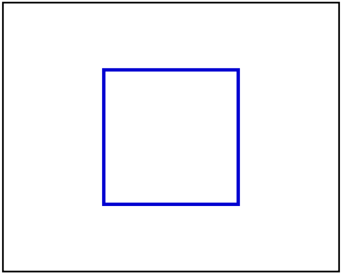

e.g. port-graph rewriting

via subterm/context and subsumption/substitution impossible